# Similarity Search

**CSE545 - Spring 2020**

Stony Brook University

H. Andrew Schwartz

$A \cap B$

# Big Data Analytics, The Class

**Goal:** Generalizations
A *model* or *summarization* of the data.

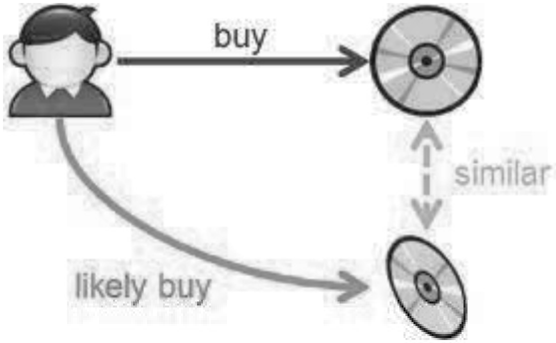
Data Frameworks

- Hadoop File System ✓
- Spark ✓
- Streaming ✓
- MapReduce ✓
- Tensorflow ✓

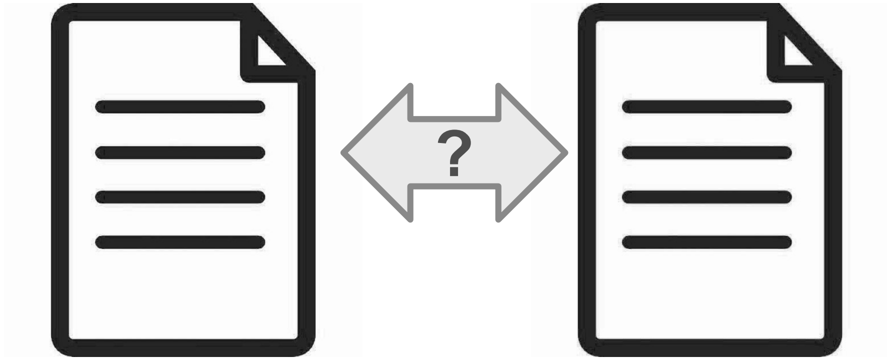
Algorithms and Analyses

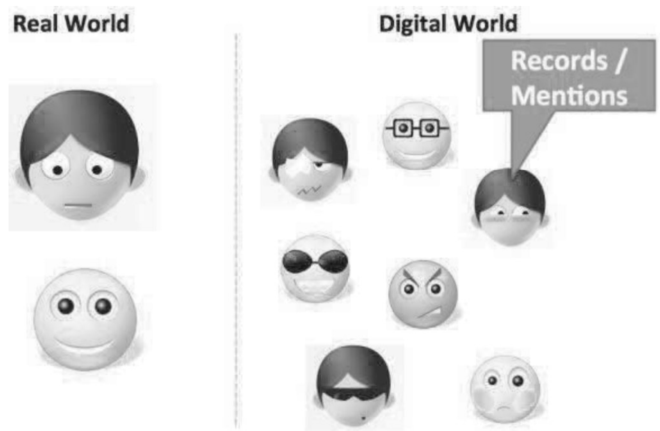
- **Similarity Search**
- Hypothesis Testing
- Link Analysis
- Recommendation Systems
- Deep Learning

# Finding Similar Items

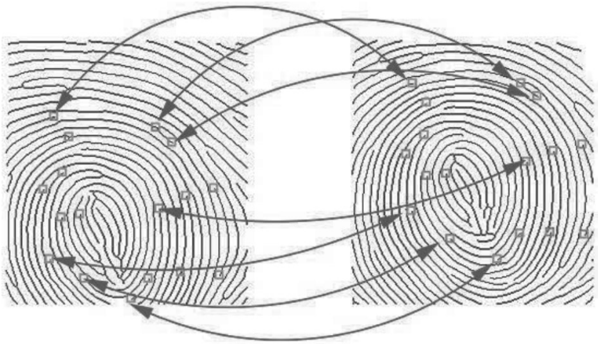



(http://blog.soton.ac.uk/hive/2012/05/10/recommendation-system-of-hive/)




(http://www.datacommunitydc.org/blog/2013/08/entity-resolution-for-big-data)



# Finding Similar Items: Topics

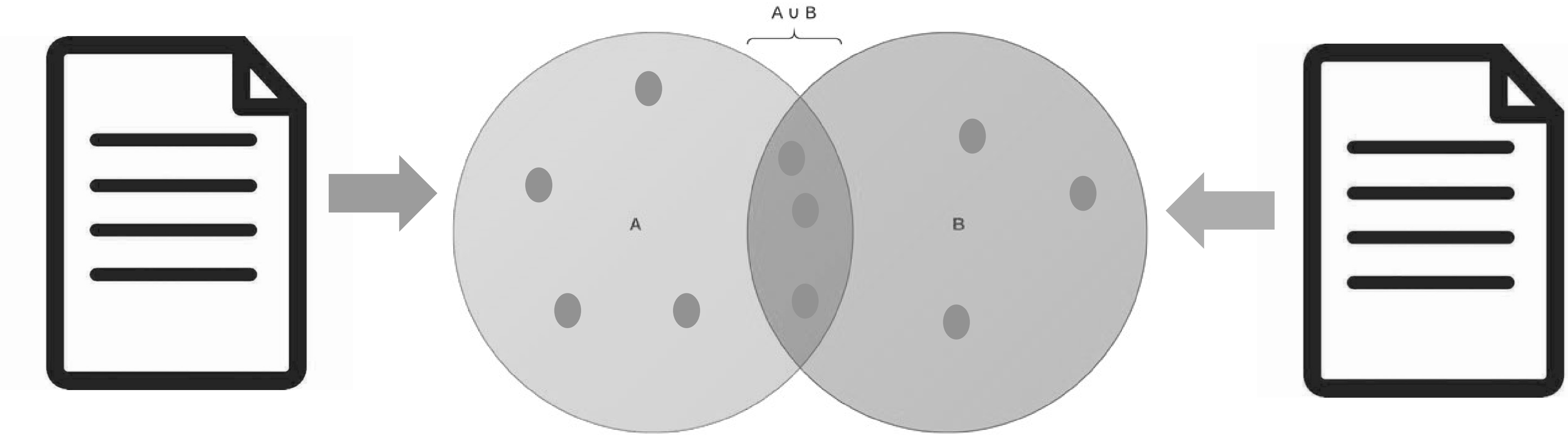
- Shingling
- Minhashing
- Locality-sensitive hashing
- Distance Metrics

# Document Similarity

**Challenge:** How to represent the document in a way that can be efficiently encoded and compared?

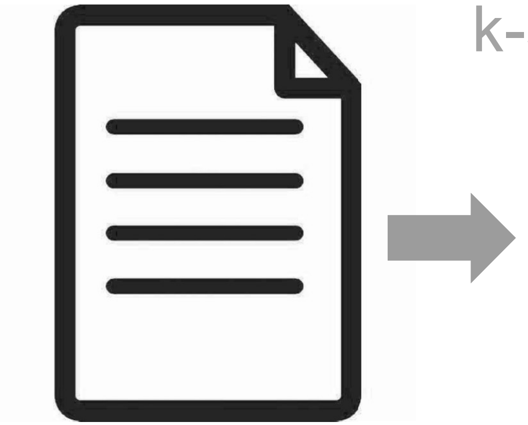
# Shingles

**Goal:** Convert documents to sets



# Shingles

**Goal:** Convert documents to sets

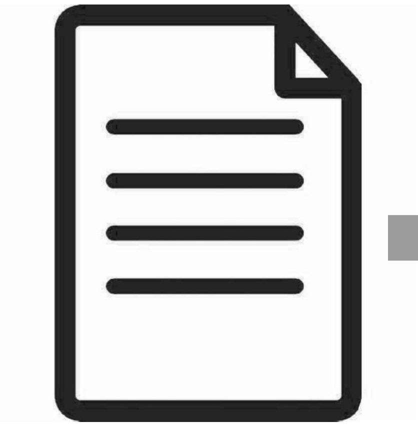


k-shingles (aka “character n-grams”)
- sequence of *k* characters

E.g. *k*=2 doc=”abcdabd”
singles(doc, 2) = {ab, bc, cd, da, bd}

# Shingles

**Goal:** Convert documents to sets



k-shingles (aka “character n-grams”)
- sequence of *k* characters

E.g. *k*=2 doc=”abcdabd”
singles(doc, 2) = {ab, bc, cd, da, bd}

- Similar documents have many common shingles
- Changing words or order has minimal effect.
- In practice use 5 < k < 10

# Shingles

**Goal:** Convert documents to sets

Large enough that any given shingle appearing a document is highly unlikely
(e.g. < .1% chance)

Can hash large shingles to smaller
(e.g. 9-shingles into 4 bytes)

*Can also use words (aka n-grams).*

s”)

acters

a, bd}

- Similar documents have many common shingles
- Changing w ds or order has minimal effect.
- **In practice use 5 < k < 10**

# Shingles

**Problem:** Even if hashing, sets of shingles are large
(e.g. 4 bytes => 4x the size of the document).

# Minhashing

**Goal:** Convert sets to shorter ids, signatures

# Minhashing

**Goal:** Convert sets to shorter ids, “signatures”

Characteristic Matrix, $X$:

| *Element* | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| $a$ | 1 | 0 | 0 | 1 |
| $b$ | 0 | 0 | 1 | 0 |
| $c$ | 0 | 1 | 0 | 1 |
| $d$ | 1 | 0 | 1 | 1 |
| $e$ | 0 | 0 | 1 | 0 |

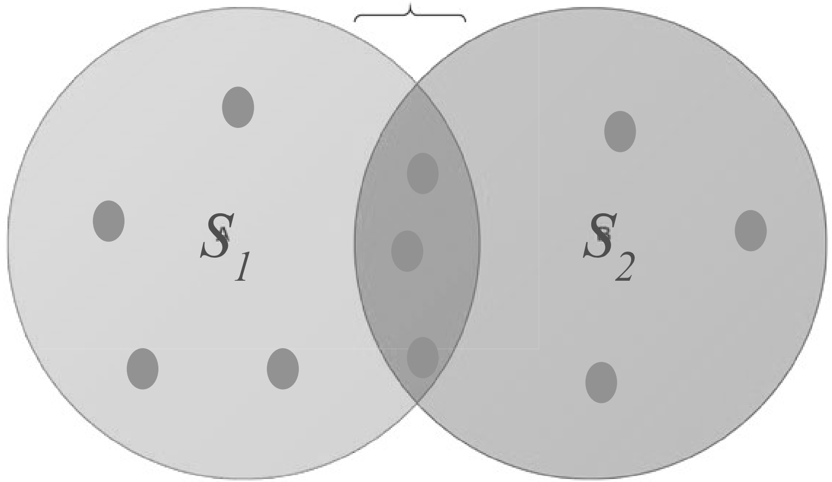
….

(Leskovec at al., 2014; http://www.mmds.org/)

often very sparse! (lots of zeros)

Jaccard Similarity:

$$sim(S_1, S_2) = \frac{S_1 \cap S_2}{S_1 \cup S_2}$$



# Minhashing

Characteristic Matrix:

| | $S_1$ | $S_2$ |
|---|---|---|
| ab | 1 | 1 |
| bc | 0 | 1 |
| de | 1 | 0 |
| ah | 1 | 1 |
| ha | 0 | 0 |
| ed | 1 | 1 |
| ca | 0 | 1 |

Jaccard Similarity:

$$sim(S_1, S_2) = \frac{S_1 \cap S_2}{S_1 \cup S_2}$$

# Minhashing

Characteristic Matrix:

| | $S_1$ | $S_2$ | |
|---|---|---|---|
| ab | 1 | 1 | * * |
| bc | 0 | 1 | * |
| de | 1 | 0 | * |
| ah | 1 | 1 | ** |
| ha | 0 | 0 | |
| ed | 1 | 1 | ** |
| ca | 0 | 1 | * |

Jaccard Similarity:

$$sim(S_1, S_2) = \frac{S_1 \cap S_2}{S_1 \cup S_2}$$

# Minhashing

Characteristic Matrix:

| | $S_1$ | $S_2$ | |
|---|---|---|---|
| ab | 1 | 1 | * * |
| bc | 0 | 1 | * |
| de | 1 | 0 | * |
| ah | 1 | 1 | ** |
| ha | 0 | 0 | |
| ed | 1 | 1 | ** |
| ca | 0 | 1 | * |

Jaccard Similarity:

$$sim(S_1, S_2) = \frac{S_1 \cap S_2}{S_1 \cup S_2}$$

$sim(S_1, S_2) = 3 / 6$

# both have / # at least one has

# Minhashing

**Problem:** Even if hashing shingle contents,
sets of shingles are large

e.g. 4 byte integer per shingle: assume all unique shingles,
=> 4x the size of the document

(since there are as many shingles as characters and 1byte per char).

# Minhashing

**Goal:** Convert sets to shorter ids, "signatures"

## Characteristic Matrix: $X$

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| ab | 1 | 0 | 1 | 0 |
| bc | 1 | 0 | 0 | 1 |
| de | 0 | 1 | 0 | 1 |
| ah | 0 | 1 | 0 | 1 |
| ha | 0 | 1 | 0 | 1 |
| ed | 1 | 0 | 1 | 0 |
| ca | 1 | 0 | 1 | 0 |

(Leskovec at al., 2014; http://www.mmds.org/)

# Minhashing

**Goal:** Convert sets to shorter ids, "signatures"

## Characteristic Matrix: $X$

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| ab | 1 | 0 | 1 | 0 |
| bc | 1 | 0 | 0 | 1 |
| de | 0 | 1 | 0 | 1 |
| ah | 0 | 1 | 0 | 1 |
| ha | 0 | 1 | 0 | 1 |
| ed | 1 | 0 | 1 | 0 |
| ca | 1 | 0 | 1 | 0 |

(Leskovec at al., 2014; http://www.mmds.org/)

**Approximate Approach:**

1) Instead of keeping whole characteristic matrix, just keep first row where 1 is encountered.

2) Shuffle and repeat to get a "signature" for each set.

# Minhashing

**Goal:** Convert sets to shorter ids, "signatures"

Characteristic Matrix: $X$

| | 1 | 3 | 1 | 2 |
|---|---|---|---|---|
| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
| ab | 1 | 0 | 1 | 0 |
| bc | 1 | 0 | 0 | 1 |
| de | 0 | 1 | 0 | 1 |
| ah | 0 | 1 | 0 | 1 |
| ha | 0 | 1 | 0 | 1 |
| ed | 1 | 0 | 1 | 0 |
| ca | 1 | 0 | 1 | 0 |

(Leskovec at al., 2014; http://www.mmds.org/)

**Approximate Approach:**

**1) Instead of keeping whole characteristic matrix, just keep first row where 1 is encountered.**

2) Shuffle and repeat to get a "signature" for each set.

# Minhashing

**Goal:** Convert sets to shorter ids, "signatures"

Characteristic Matrix: $X$

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| Signature | 1 | 3 | 1 | 2 |
| ab | 1 | 0 | 1 | 0 |
| bc | 1 | 0 | 0 | 1 |
| de | 0 | 1 | 0 | 1 |
| ah | 0 | 1 | 0 | 1 |
| ha | 0 | 1 | 0 | 1 |
| ed | 1 | 0 | 1 | 0 |
| ca | 1 | 0 | 1 | 0 |

(Leskovec at al., 2014; http://www.mmds.org/)

**Approximate Approach:**

1) Instead of keeping whole characteristic matrix, just keep first row where 1 is encountered.

**2) Shuffle and repeat to get a "signature".**

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| Signature | 2 | 1 | 2 | 1 |
| ah | 0 | 1 | 0 | 1 |
| ca | 1 | 0 | 1 | 0 |
| ed | 1 | 0 | 1 | 0 |
| de | 0 | 1 | 0 | 1 |
| ab | 1 | 0 | 1 | 0 |
| bc | 1 | 0 | 0 | 1 |

...

# Minhashing

**Goal:** Convert sets to shorter ids, "signatures"

Characteristic Matrix: $X$

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| (first row) | 1 | 3 | 1 | 2 |
| ab | 1 | 0 | 1 | 0 |
| bc | 1 | 0 | 0 | 1 |
| de | 0 | 1 | 0 | 1 |
| ah | 0 | 1 | 0 | 1 |
| ha | 0 | 1 | 0 | 1 |
| ed | 1 | 0 | 1 | 0 |
| ca | 1 | 0 | 1 | 0 |

(Leskovec at al., 2014; http://www.mmds.org/)

**Approximate Approach:**

1) Instead of keeping whole characteristic matrix, just keep first row where 1 is encountered.

**2) Shuffle and repeat to get a "signature".**

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| (first row) | 2 | 1 | 2 | 1 |
| ah | 0 | 1 | 0 | 1 |
| ca | 1 | 0 | 1 | 0 |
| ed | 1 | 0 | 1 | 0 |
| de | 0 | 1 | 0 | 1 |
| ab | 1 | 0 | 1 | 0 |
| bc | 1 | 0 | 0 | 1 |

signatures

| $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|
| 1 | 3 | 1 | 2 |
| 2 | 1 | 2 | 1 |
| ... | ... | ... | ... |

...

# Minhashing

**Goal:** Convert sets to shorter ids, “signatures”

Characteristic Matrix: $X$

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| ab | 1 | 0 | 1 | 0 |
| bc | 1 | 0 | 0 | 1 |
| de | 0 | 1 | 0 | 1 |
| ah | 0 | 1 | 0 | 1 |
| ha | 0 | 1 | 0 | 1 |
| ed | 1 | 0 | 1 | 0 |
| ca | 1 | 0 | 1 | 0 |

(Leskovec at al., 2014; http://www.mmds.org/)

**Approximate Approach:**

1) Instead of keeping whole characteristic matrix, just keep first row where 1 is encountered.

2) Shuffle and repeat to get a “signature” for each set.

**Idea:** We don’t need to actually shuffle. We can just permute row ids.

# Minhashing

Characteristic Matrix:

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| ab | 1 | 0 | 1 | 0 |
| bc | 1 | 0 | 0 | 1 |
| de | 0 | 1 | 0 | 1 |
| ah | 0 | 1 | 0 | 1 |
| ha | 0 | 1 | 0 | 1 |
| ed | 1 | 0 | 1 | 0 |
| ca | 1 | 0 | 1 | 0 |

(Leskovec at al., 2014; http://www.mmds.org/)

Minhash function: $h$

- Based on permutation of rows in the characteristic matrix, $h$ maps sets to first row where set appears.

# Minhashing

Minhash function: $h$

- Based on permutation of rows in the characteristic matrix, $h$ maps sets to first row where set appears.

Characteristic Matrix:

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| ab | 1 | 0 | 1 | 0 |
| bc | 1 | 0 | 0 | 1 |
| de | 0 | 1 | 0 | 1 |
| ah | 0 | 1 | 0 | 1 |
| ha | 0 | 1 | 0 | 1 |
| ed | 1 | 0 | 1 | 0 |
| ca | 1 | 0 | 1 | 0 |

| permuted order |
|---|
| 1 ha |
| 2 ed |
| 3 ab |
| 4 bc |
| 5 ca |
| 6 ah |
| 7 de |

(Leskovec at al., 2014; http://www.mmds.org/)

# Minhashing

Characteristic Matrix:

| | | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|---|
| 3 | ab | 1 | 0 | 1 | 0 |
| 4 | bc | 1 | 0 | 0 | 1 |
| 7 | de | 0 | 1 | 0 | 1 |
| 6 | ah | 0 | 1 | 0 | 1 |
| 1 | ha | 0 | 1 | 0 | 1 |
| 2 | ed | 1 | 0 | 1 | 0 |
| 5 | ca | 1 | 0 | 1 | 0 |

| permuted order |
|---|
| 1 ha |
| 2 ed |
| 3 ab |
| 4 bc |
| 5 ca |
| 6 ah |
| 7 de |

(Leskovec at al., 2014; http://www.mmds.org/)

Minhash function: $h$

- Based on permutation of rows in the characteristic matrix, $h$ maps sets to first row where set appears.

# Minhashing

Characteristic Matrix:

| | | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|---|
| 3 | ab | 1 | 0 | 1 | 0 |
| 4 | bc | 1 | 0 | 0 | 1 |
| 7 | de | 0 | 1 | 0 | 1 |
| 6 | ah | 0 | 1 | 0 | 1 |
| 1 | ha | 0 | 1 | 0 | 1 |
| 2 | ed | 1 | 0 | 1 | 0 |
| 5 | ca | 1 | 0 | 1 | 0 |

| permuted order |
|---|
| 1 ha |
| 2 ed |
| 3 ab |
| 4 bc |
| 5 ca |
| 6 ah |
| 7 de |

(Leskovec at al., 2014; http://www.mmds.org/)

Minhash function: $h$

- Based on permutation of rows in the characteristic matrix, $h$ maps sets to first row where set appears.

$h(S_1)$ = ed #permuted row 2
$h(S_2)$ = ha #permuted row 1
$h(S_3)$ =

# Minhashing

Characteristic Matrix:

| | | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|---|
| 3 | ab | 1 | 0 | 1 | 0 |
| 4 | bc | 1 | 0 | 0 | 1 |
| 7 | de | 0 | 1 | 0 | 1 |
| 6 | ah | 0 | 1 | 0 | 1 |
| 1 | ha | 0 | 1 | 0 | 1 |
| 2 | ed | 1 | 0 | 1 | 0 |
| 5 | ca | 1 | 0 | 1 | 0 |

| permuted order |
|---|
| 1 ha |
| 2 ed |
| 3 ab |
| 4 bc |
| 5 ca |
| 6 ah |
| 7 de |

(Leskovec at al., 2014; http://www.mmds.org/)

Minhash function: $h$

- Based on permutation of rows in the characteristic matrix, $h$ maps sets to first row where set appears.

$h(S_1)$ = ed #permuted row 2
$h(S_2)$ = ha #permuted row 1
$h(S_3)$ = ed #permuted row 2
$h(S_4)$ =

# Minhashing

Minhash function: $h$

- Based on permutation of rows in the characteristic matrix, $h$ maps sets to first row where set appears.

Characteristic Matrix:

| | | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|---|
| 3 | ab | 1 | 0 | 1 | 0 |
| 4 | bc | 1 | 0 | 0 | 1 |
| 7 | de | 0 | 1 | 0 | 1 |
| 6 | ah | 0 | 1 | 0 | 1 |
| 1 | ha | 0 | 1 | 0 | 1 |
| 2 | ed | 1 | 0 | 1 | 0 |
| 5 | ca | 1 | 0 | 1 | 0 |

| permuted order |
|---|
| 1 ha |
| 2 ed |
| 3 ab |
| 4 bc |
| 5 ca |
| 6 ah |
| 7 de |

(Leskovec at al., 2014; http://www.mmds.org/)

$h(S_1)$ = ed #permuted row 2
$h(S_2)$ = ha #permuted row 1
$h(S_3)$ = ed #permuted row 2
$h(S_4)$ = ha #permuted row 1

# Minhashing

Characteristic Matrix:

| | | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|---|
| 3 | ab | 1 | 0 | 1 | 0 |
| 4 | bc | 1 | 0 | 0 | 1 |
| 7 | de | 0 | 1 | 0 | 1 |
| 6 | ah | 0 | 1 | 0 | 1 |
| 1 | ha | 0 | 1 | 0 | 1 |
| 2 | ed | 1 | 0 | 1 | 0 |
| 5 | ca | 1 | 0 | 1 | 0 |

(Leskovec at al., 2014; http://www.mmds.org/)

Minhash function: $h$

- Based on permutation of rows in the characteristic matrix, $h$ maps sets to rows.

Signature matrix: *M*

- Record first row where each set had a 1 in the given permutation

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| $h_1$ | 2 | 1 | 2 | 1 |

$h_1(S_1)$ = ed #permuted row 2
$h_1(S_2)$ = ha #permuted row 1
$h_1(S_3)$ = ed #permuted row 2
$h_1(S_4)$ = ha #permuted row 1

# Minhashing

Characteristic Matrix:

| | | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|---|
| 3 | ab | 1 | 0 | 1 | 0 |
| 4 | bc | 1 | 0 | 0 | 1 |
| 7 | de | 0 | 1 | 0 | 1 |
| 6 | ah | 0 | 1 | 0 | 1 |
| 1 | ha | 0 | 1 | 0 | 1 |
| 2 | ed | 1 | 0 | 1 | 0 |
| 5 | ca | 1 | 0 | 1 | 0 |

(Leskovec at al., 2014; http://www.mmds.org/)

Minhash function: $h$

- Based on permutation of rows in the characteristic matrix, $h$ maps sets to rows.

Signature matrix: $M$

- Record first row where each set had a 1 in the given permutation

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| $h_1$ | 2 | 1 | 2 | 1 |

$h_1(S_1)$ = ed #permuted row 2

$h_1(S_2)$ = ha #permuted row 1

$h_1(S_3)$ = ed #permuted row

# Minhashing

Characteristic Matrix:

| | | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|---|
| 3 | ab | 1 | 0 | 1 | 0 |
| 4 | bc | 1 | 0 | 0 | 1 |
| 7 | de | 0 | 1 | 0 | 1 |
| 6 | ah | 0 | 1 | 0 | 1 |
| 1 | ha | 0 | 1 | 0 | 1 |
| 2 | ed | 1 | 0 | 1 | 0 |
| 5 | ca | 1 | 0 | 1 | 0 |

(Leskovec at al., 2014; http://www.mmds.org/)

Minhash function: $h$

- Based on permutation of rows in the characteristic matrix, $h$ maps sets to rows.

Signature matrix: *M*

- Record first row where each set had a 1 in the given permutation

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| $h_1$ | 2 | 1 | 2 | 1 |

$h_1(S_1)$ = ed #permuted row

2

$h_1(S_2)$ = ha #permuted row

1

$h_1(S_3)$ = ed #permuted row

# Minhashing

Characteristic Matrix:

| | | | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|---|---|
| 4 | 3 | ab | 1 | 0 | 1 | 0 |
| 2 | 4 | bc | 1 | 0 | 0 | 1 |
| 1 | 7 | de | 0 | 1 | 0 | 1 |
| 3 | 6 | ah | 0 | 1 | 0 | 1 |
| 6 | 1 | ha | 0 | 1 | 0 | 1 |
| 7 | 2 | ed | 1 | 0 | 1 | 0 |
| 5 | 5 | ca | 1 | 0 | 1 | 0 |

(Leskovec at al., 2014; http://www.mmds.org/)

Minhash function: $h$

- Based on permutation of rows in the characteristic matrix, $h$ maps sets to rows.

Signature matrix: *M*

- Record first row where each set had a 1 in the given permutation

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| $h_1$ | 2 | 1 | 2 | 1 |
| $h_2$ | | | | |

# Minhashing

Characteristic Matrix:

| | | | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|---|---|
| 4 | 3 | ab | 1 | 0 | 1 | 0 |
| 2 | 4 | bc | 1 | 0 | 0 | 1 |
| 1 | 7 | de | 0 | 1 | 0 | 1 |
| 3 | 6 | ah | 0 | 1 | 0 | 1 |
| 6 | 1 | ha | 0 | 1 | 0 | 1 |
| 7 | 2 | ed | 1 | 0 | 1 | 0 |
| 5 | 5 | ca | 1 | 0 | 1 | 0 |

(Leskovec at al., 2014; http://www.mmds.org/)

Minhash function: $h$

- Based on permutation of rows in the characteristic matrix, $h$ maps sets to rows.

Signature matrix: *M*

- Record first row where each set had a 1 in the given permutation

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| $h_1$ | 2 | 1 | 2 | 1 |
| $h_2$ | 2 | 1 | 4 | 1 |

# Minhashing

Characteristic Matrix:

| | | | | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|---|---|---|
| 1 | 4 | 3 | ab | 1 | 0 | 1 | 0 |
| 3 | 2 | 4 | bc | 1 | 0 | 0 | 1 |
| 7 | 1 | 7 | de | 0 | 1 | 0 | 1 |
| 6 | 3 | 6 | ah | 0 | 1 | 0 | 1 |
| 2 | 6 | 1 | ha | 0 | 1 | 0 | 1 |
| 5 | 7 | 2 | ed | 1 | 0 | 1 | 0 |
| 4 | 5 | 5 | ca | 1 | 0 | 1 | 0 |

(Leskovec at al., 2014; http://www.mmds.org/)

Minhash function: $h$

- Based on permutation of rows in the characteristic matrix, $h$ maps sets to rows.

Signature matrix: *M*

- Record first row where each set had a 1 in the given permutation

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| $h_1$ | 2 | 1 | 2 | 1 |
| $h_2$ | 2 | 1 | 4 | 1 |
| $h_3$ | | | | |

# Minhashing

Characteristic Matrix:

| | | | | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|---|---|---|
| 1 | 4 | 3 | ab | 1 | 0 | 1 | 0 |
| 3 | 2 | 4 | bc | 1 | 0 | 0 | 1 |
| 7 | 1 | 7 | de | 0 | 1 | 0 | 1 |
| 6 | 3 | 6 | ah | 0 | 1 | 0 | 1 |
| 2 | 6 | 1 | ha | 0 | 1 | 0 | 1 |
| 5 | 7 | 2 | ed | 1 | 0 | 1 | 0 |
| 4 | 5 | 5 | ca | 1 | 0 | 1 | 0 |

(Leskovec at al., 2014; http://www.mmds.org/)

Minhash function: $h$

- Based on permutation of rows in the characteristic matrix, $h$ maps sets to rows.

Signature matrix: *M*

- Record first row where each set had a 1 in the given permutation

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| $h_1$ | 2 | 1 | 2 | 1 |
| $h_2$ | 2 | 1 | 4 | 1 |
| $h_3$ | 1 | 2 | 1 | 2 |

# Minhashing

Characteristic Matrix:

| | | | | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|---|---|---|
| 1 | 4 | 3 | ab | 1 | 0 | 1 | 0 |
| 3 | 2 | 4 | bc | 1 | 0 | 0 | 1 |
| 7 | 1 | 7 | de | 0 | 1 | 0 | 1 |
| 6 | 3 | 6 | ah | 0 | 1 | 0 | 1 |
| 2 | 6 | 1 | ha | 0 | 1 | 0 | 1 |
| 5 | 7 | 2 | ed | 1 | 0 | 1 | 0 |
| 4 | 5 | 5 | ca | 1 | 0 | 1 | 0 |

(Leskovec at al., 2014; http://www.mmds.org/)

Minhash function: $h$

- Based on permutation of rows in the characteristic matrix, $h$ maps sets to rows.

Signature matrix: *M*

- Record first row where each set had a 1 in the given permutation

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| $h_1$ | 2 | 1 | 2 | 1 |
| $h_2$ | 2 | 1 | 4 | 1 |
| $h_3$ | 1 | 2 | 1 | 2 |
| … | | | | |
| … | | | | |

# Minhashing

**Property of signature matrix:**
The probability for any $h_i$ (i.e. any row), that $h_i(S_1) = h_i(S_2)$ is the same as Sim($S_1$, $S_2$)

Characteristic Matrix:

| | | | | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|---|---|---|
| 1 | 4 | 3 | ab | 1 | 0 | 1 | 0 |
| 3 | 2 | 4 | bc | 1 | 0 | 0 | 1 |
| 7 | 1 | 7 | de | 0 | 1 | 0 | 1 |
| 6 | 3 | 6 | ah | 0 | 1 | 0 | 1 |
| 2 | 6 | 1 | ha | 0 | 1 | 0 | 1 |
| 5 | 7 | 2 | ed | 1 | 0 | 1 | 0 |
| 4 | 5 | 5 | ca | 1 | 0 | 1 | 0 |

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| $h_1$ | 2 | 1 | 2 | 1 |
| $h_2$ | 2 | 1 | 4 | 1 |
| $h_3$ | 1 | 2 | 1 | 2 |
| ... | | | | |
| ... | | | | |

(Leskovec at al., 2014; http://www.mmds.org/)

# Minhashing

**Property of signature matrix:**
The probability for any $h_i$ (i.e. any row), that $h_i(S_1) = h_i(S_2)$ is the same as Sim($S_1$, $S_2$)

Thus, similarity of signatures $S_1$, $S_2$ is the fraction of minhash functions (i.e. rows) in which they agree.

Characteristic Matrix:

| | | | | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|---|---|---|
| 1 | 4 | 3 | ab | 1 | 0 | 1 | 0 |
| 3 | 2 | 4 | bc | 1 | 0 | 0 | 1 |
| 7 | 1 | 7 | de | 0 | 1 | 0 | 1 |
| 6 | 3 | 6 | ah | 0 | 1 | 0 | 1 |
| 2 | 6 | 1 | ha | 0 | 1 | 0 | 1 |
| 5 | 7 | 2 | ed | 1 | 0 | 1 | 0 |
| 4 | 5 | 5 | ca | 1 | 0 | 1 | 0 |

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| $h_1$ | 2 | 1 | 2 | 1 |
| $h_2$ | 2 | 1 | 4 | 1 |
| $h_3$ | 1 | 2 | 1 | 2 |
| … | | | | |
| … | | | | |

(Leskovec at al., 2014; http://www.mmds.org/)

# Minhashing

Characteristic Matrix:

| | | | | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|---|---|---|
| 1 | 4 | 3 | ab | 1 | 0 | | 0 |
| 3 | 2 | | | | | | |
| 7 | | | | | | | |
| 6 | 3 | 6 | ah | 0 | | | |
| 2 | 6 | 1 | ha | 0 | 1 | 0 | 1 |
| 5 | 7 | 2 | ed | 1 | 0 | 1 | 0 |
| 4 | 5 | 5 | ca | 1 | 0 | 1 | 0 |

**Property of signature matrix:**
The probability for any $h_i$ (i.e. any row), that $h_i(S_1) = h_i(S_2)$ is the same as Sim($S_1$, $S_2$)

Thus, similarity of signatures $S_1$, $S_2$ is the fraction of minhash functions (i.e. rows) in which they agree.

Estimate with a random sample of permutations (i.e. ~100)

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| $h_1$ | 2 | 1 | 2 | 1 |
| $h_2$ | 2 | 1 | 4 | 1 |
| $h_3$ | 1 | 2 | 1 | 2 |
| … | | | | |
| … | | | | |

(Leskovec at al., 2014; http://www.mmds.org/)

# Minhashing

**Property of signature matrix:**
The probability for any $h_i$ (i.e. any row), that $h_i(S_1) = h_i(S_2)$ is the same as Sim($S_1$, $S_2$)

Thus, similarity of signatures $S_1$, $S_2$ is the fraction of minhash functions (i.e. rows) in which they agree.

Characteristic Matrix:

| | | | | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|---|---|---|
| 1 | 4 | 3 | ab | 1 | 0 | [illegible] | 0 |
| 3 | 2 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 7 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 6 | 3 | 6 | ah | 0 | [illegible] | [illegible] | [illegible] |
| 2 | 6 | 1 | ha | 0 | 1 | 0 | 1 |
| 5 | 7 | 2 | ed | 1 | 0 | 1 | 0 |
| 4 | 5 | 5 | ca | 1 | 0 | 1 | 0 |

Estimate with a random sample of permutations (i.e. ~100)

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| $h_1$ | 2 | 1 | 2 | 1 |
| $h_2$ | 2 | 1 | 4 | 1 |
| $h_3$ | 1 | 2 | 1 | 2 |

Estimated Sim($S_1$, $S_3$) = agree / all = 2/3

(Leskovec at al., 2014; http://www.mmds.org/)

# Minhashing

**Property of signature matrix:**
The probability for any $h_i$ (i.e. any row), that $h_i(S_1) = h_i(S_2)$ is the same as Sim($S_1$, $S_2$)

Thus, similarity of signatures $S_1$, $S_2$ is the fraction of minhash functions (i.e. rows) in which they agree.

Characteristic Matrix:

| | | | | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|---|---|---|
| 1 | 4 | 3 | ab | 1 | 0 | 1 | 0 |
| 3 | 2 | 4 | bc | 1 | 0 | 0 | 1 |
| 7 | 1 | 7 | de | 0 | 1 | 0 | 1 |
| 6 | 3 | 6 | ah | 0 | 1 | 0 | 1 |
| 2 | 6 | 1 | ha | 0 | 1 | 0 | 1 |
| 5 | 7 | 2 | ed | 1 | 0 | 1 | 0 |
| 4 | 5 | 5 | ca | 1 | 0 | 1 | 0 |

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| $h_1$ | 2 | 1 | 2 | 1 |
| $h_2$ | 2 | 1 | 4 | 1 |
| $h_3$ | 1 | 2 | 1 | 2 |

Estimated Sim($S_1$, $S_3$) = agree / all = 2/3

Real Sim($S_1$, $S_3$) = Type a / (a + b + c) = 3/4

(Leskovec at al., 2014; http://www.mmds.org/)

# Minhashing

**Property of signature matrix:**
The probability for any $h_i$ (i.e. any row), that $h_i(S_1) = h_i(S_2)$ is the same as Sim($S_1$, $S_2$)

Thus, similarity of signatures $S_1$, $S_2$ is the fraction of minhash functions (i.e. rows) in which they agree.

Characteristic Matrix:

| | | | | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|---|---|---|
| 1 | 4 | 3 | ab | 1 | 0 | 1 | 0 |
| 3 | 2 | 4 | bc | 1 | 0 | 0 | 1 |
| 7 | 1 | 7 | de | 0 | 1 | 0 | 1 |
| 6 | 3 | 6 | ah | 0 | 1 | 0 | 1 |
| 2 | 6 | 1 | ha | 0 | 1 | 0 | 1 |
| 5 | 7 | 2 | ed | 1 | 0 | 1 | 0 |
| 4 | 5 | 5 | ca | 1 | 0 | 1 | 0 |

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| $h_1$ | 2 | 1 | 2 | 1 |
| $h_2$ | 2 | 1 | 4 | 1 |
| $h_3$ | 1 | 2 | 1 | 2 |

Estimated Sim($S_1$, $S_3$) = agree / all = 2/3

Real Sim($S_1$, $S_3$) = Type a / (a + b + c) = 3/4

Try Sim($S_2$, $S_4$) and Sim($S_1$, $S_2$)

(Leskovec at al., 2014; http://www.mmds.org/)

# Minhashing

**Error Bound?**

Characteristic Matrix:

| | | | | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|---|---|---|
| 1 | 4 | 3 | ab | 1 | 0 | 1 | 0 |
| 3 | 2 | 4 | bc | 1 | 0 | 0 | 1 |
| 7 | 1 | 7 | de | 0 | 1 | 0 | 1 |
| 6 | 3 | 6 | ah | 0 | 1 | 0 | 1 |
| 2 | 6 | 1 | ha | 0 | 1 | 0 | 1 |
| 5 | 7 | 2 | ed | 1 | 0 | 1 | 0 |
| 4 | 5 | 5 | ca | 1 | 0 | 1 | 0 |

(Leskovec at al., 2014; http://www.mmds.org/)

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| $h_1$ | 2 | 1 | 2 | 1 |
| $h_2$ | 2 | 1 | 4 | 1 |
| $h_3$ | 1 | 2 | 1 | 2 |

Estimated Sim($S_1$, $S_3$) = agree / all = 2/3

Real Sim($S_1$, $S_3$) = Type a / (a + b + c) = 3/4

Try Sim($S_2$, $S_4$) and Sim($S_1$, $S_2$)

# Minhashing

Characteristic Matrix:

| | | | | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|---|---|---|
| 1 | 4 | 3 | ab | 1 | 0 | 1 | 0 |
| 3 | 2 | 4 | bc | 1 | 0 | 0 | 1 |
| 7 | 1 | 7 | de | 0 | 1 | 0 | 1 |
| 6 | 3 | 6 | ah | 0 | 1 | 0 | 1 |
| 2 | 6 | 1 | ha | 0 | 1 | 0 | 1 |
| 5 | 7 | 2 | ed | 1 | 0 | 1 | 0 |
| 4 | 5 | 5 | ca | 1 | 0 | 1 | 0 |

(Leskovec at al., 2014; http://www.mmds.org/)

**Error Bound?**

Expect error: **O(*1*/√*k*)** (k hashes)

Why? Each row is a random observation of 1 or 0 (match or not) with *P(match=1)* = *Sim(S1, S2).*

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| $h_1$ | 2 | 1 | 2 | 1 |
| $h_2$ | 2 | 1 | 4 | 1 |
| $h_3$ | 1 | 2 | 1 | 2 |

Estimated Sim($S_1$, $S_3$) = agree / all = 2/3

Real Sim($S_1$, $S_3$) = Type a / (a + b + c) = 3/4

Try Sim($S_2$, $S_4$) and Sim($S_1$, $S_2$)

# Minhashing

Characteristic Matrix:

| | | | | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|---|---|---|
| 1 | 4 | 3 | ab | 1 | 0 | 1 | 0 |
| 3 | 2 | 4 | bc | 1 | 0 | 0 | 1 |
| 7 | 1 | 7 | de | 0 | 1 | 0 | 1 |
| 6 | 3 | 6 | ah | 0 | 1 | 0 | 1 |
| 2 | 6 | 1 | ha | 0 | 1 | 0 | 1 |
| 5 | 7 | 2 | ed | 1 | 0 | 1 | 0 |
| 4 | 5 | 5 | ca | 1 | 0 | 1 | 0 |

(Leskovec at al., 2014; http://www.mmds.org/)

**Error Bound?**

Expect error: **O(*1*/√*k*)** (k hashes)

Why? Each row is a random observation of 1 or 0 (match or not) with *P(match=1) = Sim(S1, S2).*

N = *k* observations

Standard deviation(*std*)? < 1 (worst case is 0.5)

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| $h_1$ | 2 | 1 | 2 | 1 |
| $h_2$ | 2 | 1 | 4 | 1 |
| $h_3$ | 1 | 2 | 1 | 2 |

Estimated Sim($S_1$, $S_3$) = agree / all = 2/3

Real Sim($S_1$, $S_3$) = Type a / (a + b + c) = 3/4

Try Sim($S_2$, $S_4$) and Sim($S_1$, $S_2$)

# Minhashing

Characteristic Matrix:

| | | | | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|---|---|---|
| 1 | 4 | 3 | ab | 1 | 0 | 1 | 0 |
| 3 | 2 | 4 | bc | 1 | 0 | 0 | 1 |
| 7 | 1 | 7 | de | 0 | 1 | 0 | 1 |
| 6 | 3 | 6 | ah | 0 | 1 | 0 | 1 |
| 2 | 6 | 1 | ha | 0 | 1 | 0 | 1 |
| 5 | 7 | 2 | ed | 1 | 0 | 1 | 0 |
| 4 | 5 | 5 | ca | 1 | 0 | 1 | 0 |

(Leskovec at al., 2014; http://www.mmds.org/)

**Error Bound?**

Expect error: **O(*1*/√*k*)** (k hashes)

Why? Each row is a random observation of 1 or 0 (match or not) with *P(match=1) = Sim(S1, S2).*

N = *k* observations

Standard deviation(*std*)? < 1 (worst case is 0.5)

Standard Error of Mean = *std*/√N

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|
| $h_1$ | 2 | 1 | 2 | 1 |
| $h_2$ | 2 | 1 | 4 | 1 |
| $h_3$ | 1 | 2 | 1 | 2 |

Estimated Sim($S_1$, $S_3$) = agree / all = 2/3

Real Sim($S_1$, $S_3$) = Type a / (a + b + c) = 3/4

Try Sim($S_2$, $S_4$) and Sim($S_1$, $S_2$)

# Minhashing

In Practice

Problem:

- Can’t reasonably do permutations (huge space)
- Can’t randomly grab rows according to an order (random disk seeks = slow!)

# Minhashing

In Practice

Problem:

- Can’t reasonably do permutations (huge space)
- Can’t randomly grab rows according to an order (random disk seeks = slow!)

Solution: Use “random” hash functions.

- Setup:
    - Pick ~100 hash functions, hashes
    - Store M[i][s] = a potential minimum $h_i(r)$ *#initialized to infinity (num hashs x num sets)*

# Minhashing

Solution: Use "random" hash functions.

Setup:

```
hashes = [getHfunc(i) for i in rand(1, num=100)]
                                    #100 hash functions, seeded random
for i in hashes: for s in sets:
    Sig[i][s] = np.inf #represents a potential minimum h_i(r) ; initially infinity
```

# Minhashing

Solution: Use "random" hash functions.

Setup:

```
hashes = [getHfunc(i) for i in rand(1, num=100)]
                                     #100 hash functions, seeded random
for i in hashes: for s in sets:
   Sig[i][s] = np.inf #represents a potential minimum h_i(r) ; initially infinity
```

Algorithm ("efficient minhashing"):

```
for r in rows of cm: #cm is characteristic matrix
 compute h_i(r) for all i in hashes #precompute 100 values
 for each set s in sets: #columns of cm
   if cm[r][s] == 1:
     for i in hashes: #check which hash produces smallest value
       if h_i(r) < Sig[i][s]: Sig[i][s] = h_i(r)
```

# Minhashing

Solution: Use “random” hash functions.

Setup:

hashes = [getHfunc(i) for i in rand(1, num=100)]

*#100 hash functions, seeded random*

for i in hashes: for s in sets:

Sig[i][s] = np.inf *#represents a potential minimum $h_i(r)$ ; initially infinity*

Algorithm (“efficient minhashing”) without charact matrix:

```
for feat in shins: #shins is all unique shingles
 compute h_i(feat) for all i in hashes #precompute 100 values
 for each set s in sets: #sets is list of shingle sets
   if feat in s:
     for i in hashes: #check which hash produces smallest value
       if h_i(feat) < Sig[i][s_id]: Sig[i][s_id] = h_i(feat)
```

# Minhashing

**Problem:** Even if hashing, sets of shingles are large (e.g. 4 bytes => 4x the size of the document).

# Minhashing

**Problem:** Even if hashing, sets of shingles are large (e.g. 4 bytes => 4x the size of the document).

**New Problem:** Even if the size of signatures are small, it can be computationally expensive to find similar pairs.

E.g. 1m documents; 1,000,000 choose 2 = 500,000,000,000 pairs!

# Minhashing

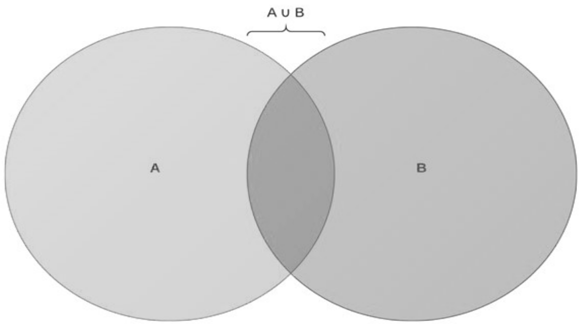
**Problem:** Even if hashing, sets of shingles are large (e.g. 4 bytes => 4x the size of the document).

**New Problem:** Even if the size of signatures are small, it can be computationally expensive to find similar pairs.

E.g. 1m documents; 1,000,000 choose 2 = 500,000,000,000 pairs!

(1m documents isn’t even “big data”)

# Document Similarity




Duplicate web pages (useful for ranking

Plagiarism

Cluster News Articles

Anything similar to documents: movie/music/art tastes, product characteristics

COVID-19 Report matching

# Locality-Sensitive Hashing

**Goal:** find pairs of minhashes ***likely*** to be similar (in order to then test more precisely for similarity).

Candidate pairs: pairs of elements to be evaluated for similarity.

# Locality-Sensitive Hashing

**Goal:** find pairs of minhashes ***likely*** to be similar (in order to then test more precisely for similarity).

Candidate pairs: pairs of elements to be evaluated for similarity.

If we wanted the similarity for all pairs of documents, could anything be done?

# Locality-Sensitive Hashing

**Goal:** find pairs of minhashes likely to be similar (in order to then test more precisely for similarity).

Candidate pairs: pairs of elements to be evaluated for similarity.

Approach: Hash multiple times over subsets of data: similar items are likely in the same bucket once.

# Locality-Sensitive Hashing

**Goal:** find pairs of minhashes likely to be similar (in order to then test more precisely for similarity).

Candidate pairs: pairs of elements to be evaluated for similarity.

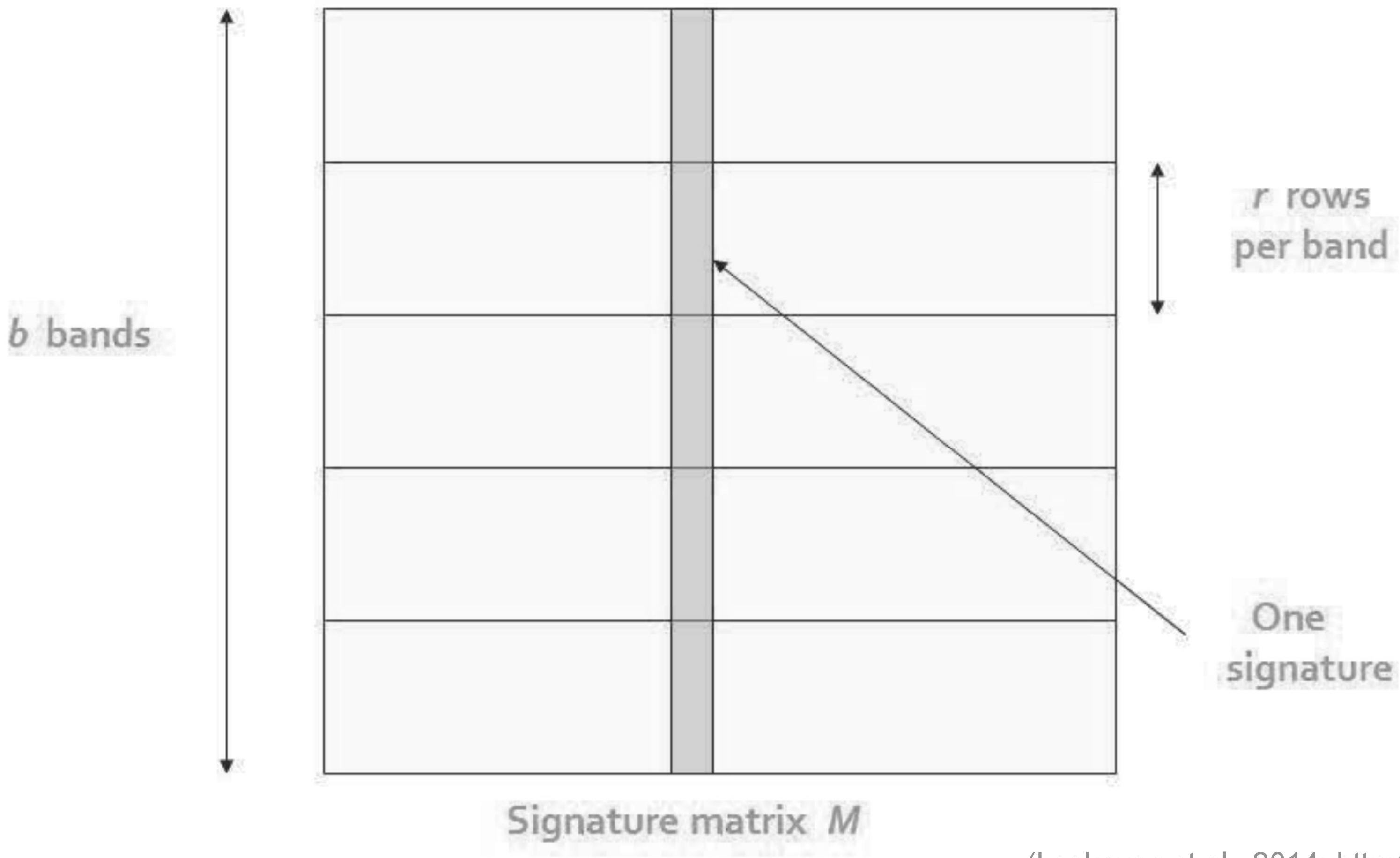
Approach: Hash multiple times over subsets of data: similar items are likely in the same bucket once.

Approach from MinHash: Hash columns of signature matrix

⟹ Candidate pairs end up in the same bucket.

# Locality-Sensitive Hashing

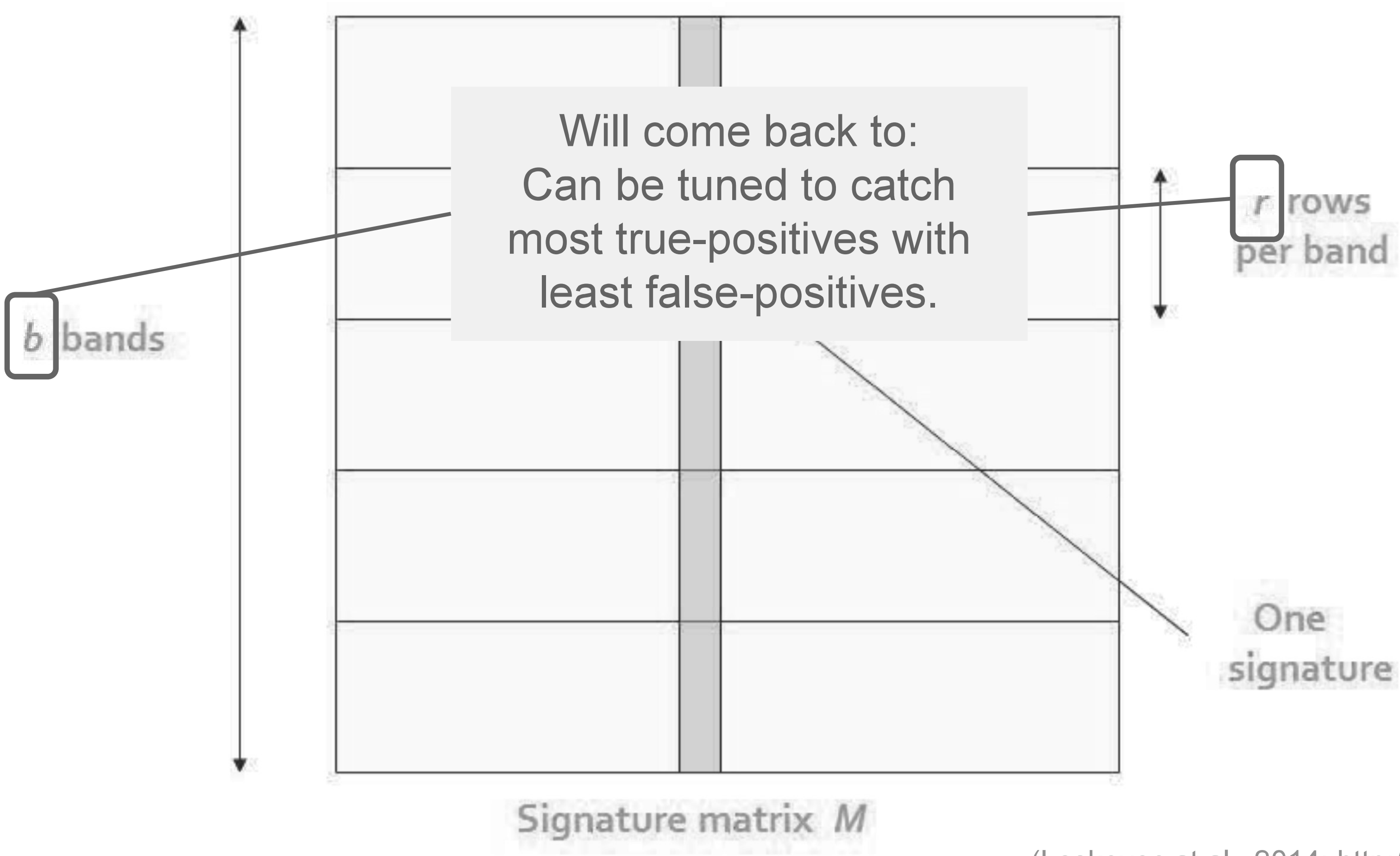
**Step 1: Divide signature matrix into *b* bands**




(Leskovec at al., 2014; http://www.mmds.org/)

# Locality-Sensitive Hashing

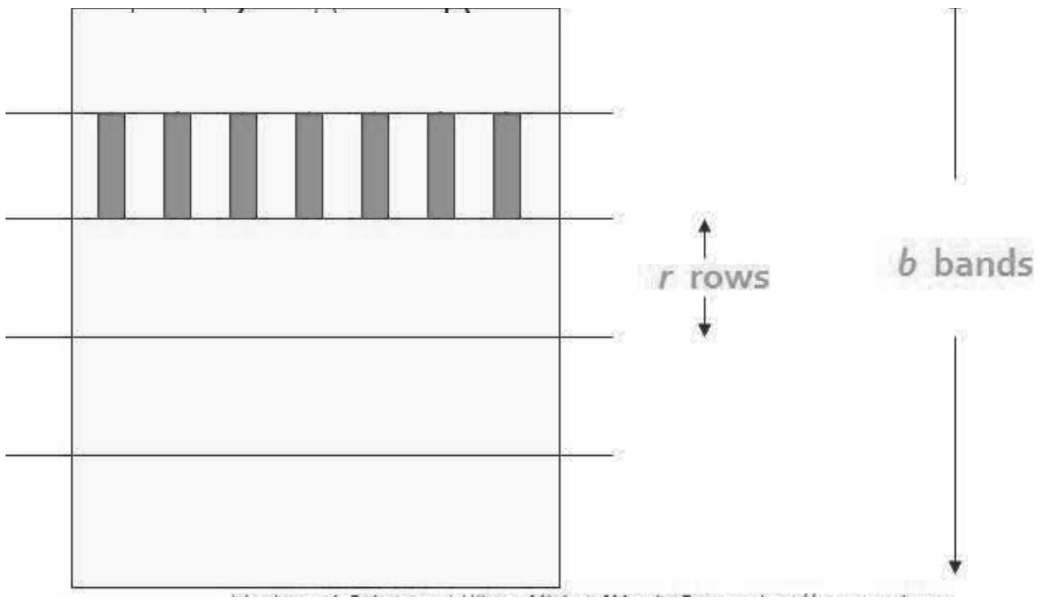
## Step 1: Divide into *b* bands




(Leskovec at al., 2014; http://www.mmds.org/)

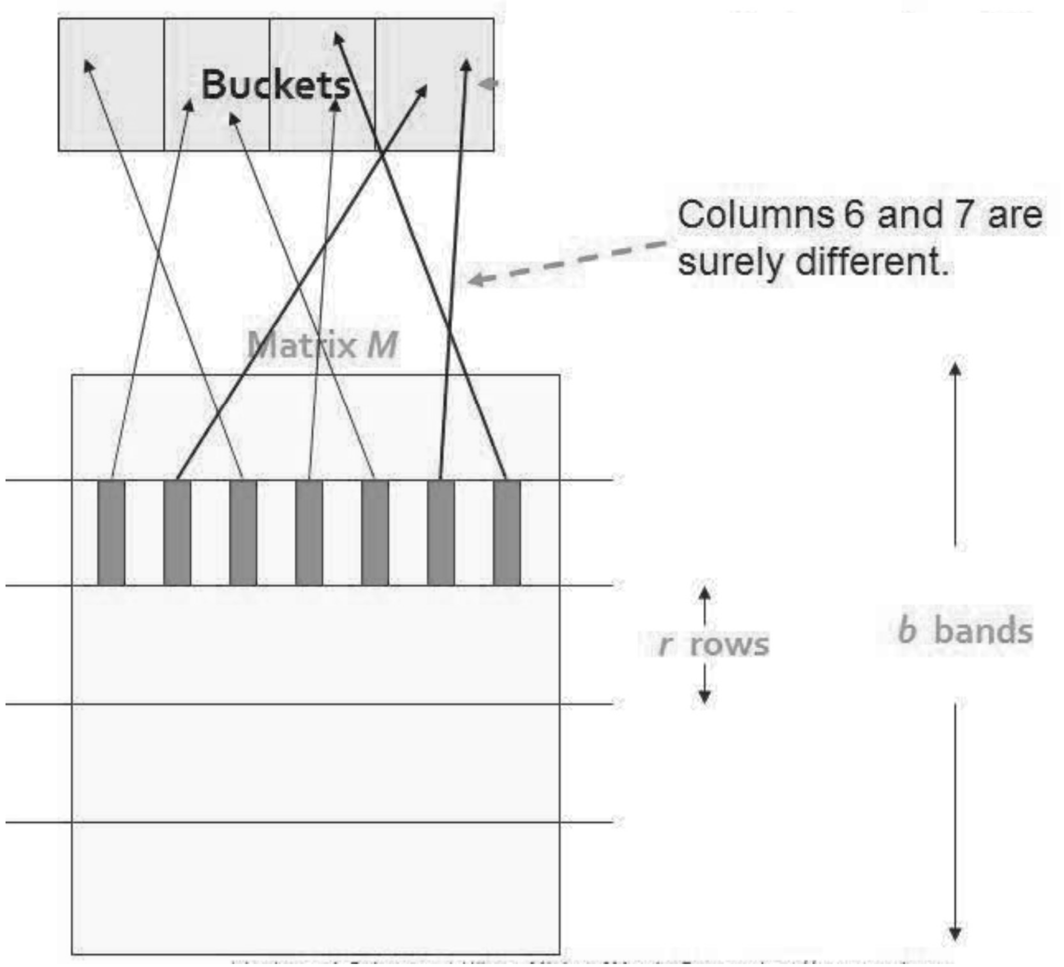
# Locality-Sensitive Hashing

**Step 1: Divide into *b* bands**
**Step 2: Hash columns within bands (one hash per band)**




(Leskovec at al., 2014; http://www.mmds.org/)

# Locality-Sensitive Hashing

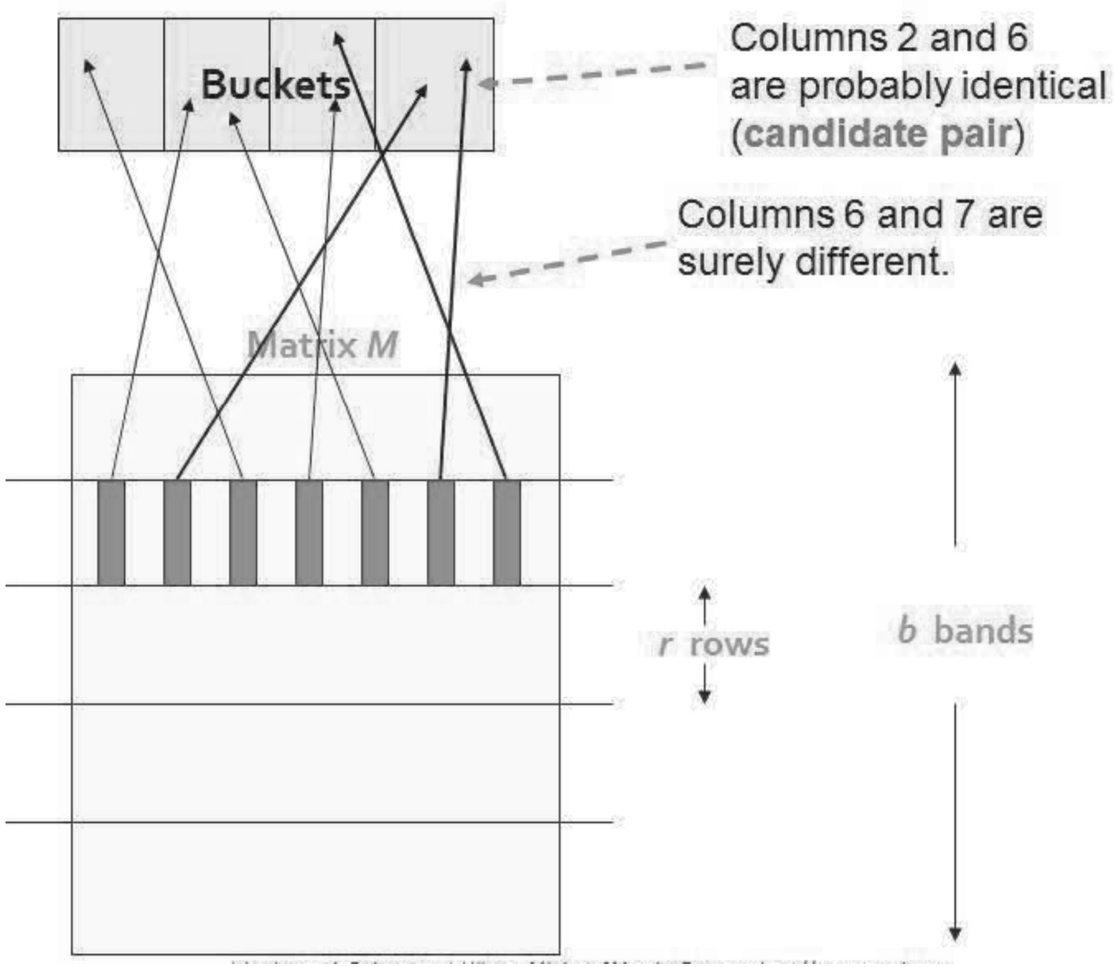



**Step 1: Divide into *b* bands**
**Step 2: Hash columns within bands (one hash per band)**

(Leskovec at al., 2014; http://www.mmds.org/)

# Locality-Sensitive Hashing

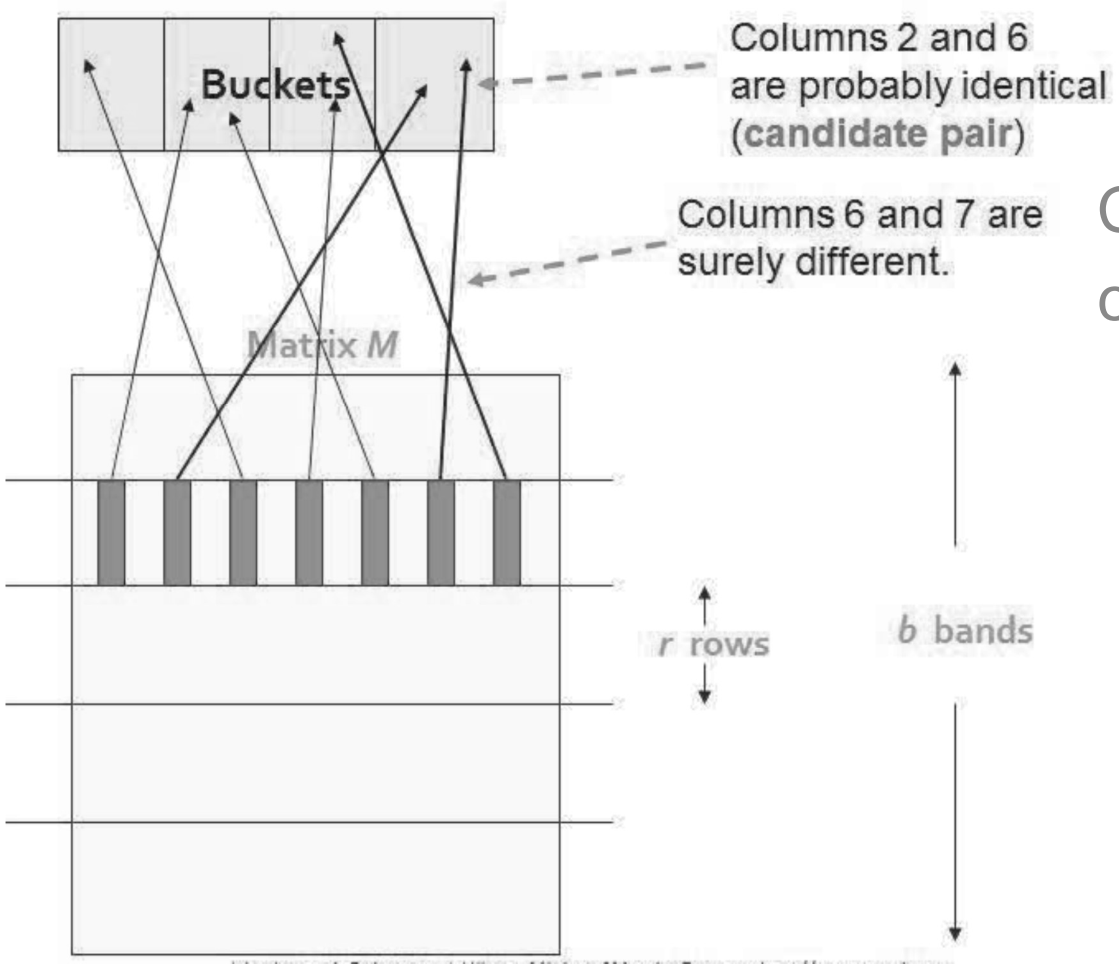
**Step 1: Divide into *b* bands**
**Step 2: Hash columns within bands (one hash per band)**




(Leskovec at al., 2014; http://www.mmds.org/)

# Locality-Sensitive Hashing

**Step 1: Divide into *b* bands**
**Step 2: Hash columns within bands (one hash per band)**

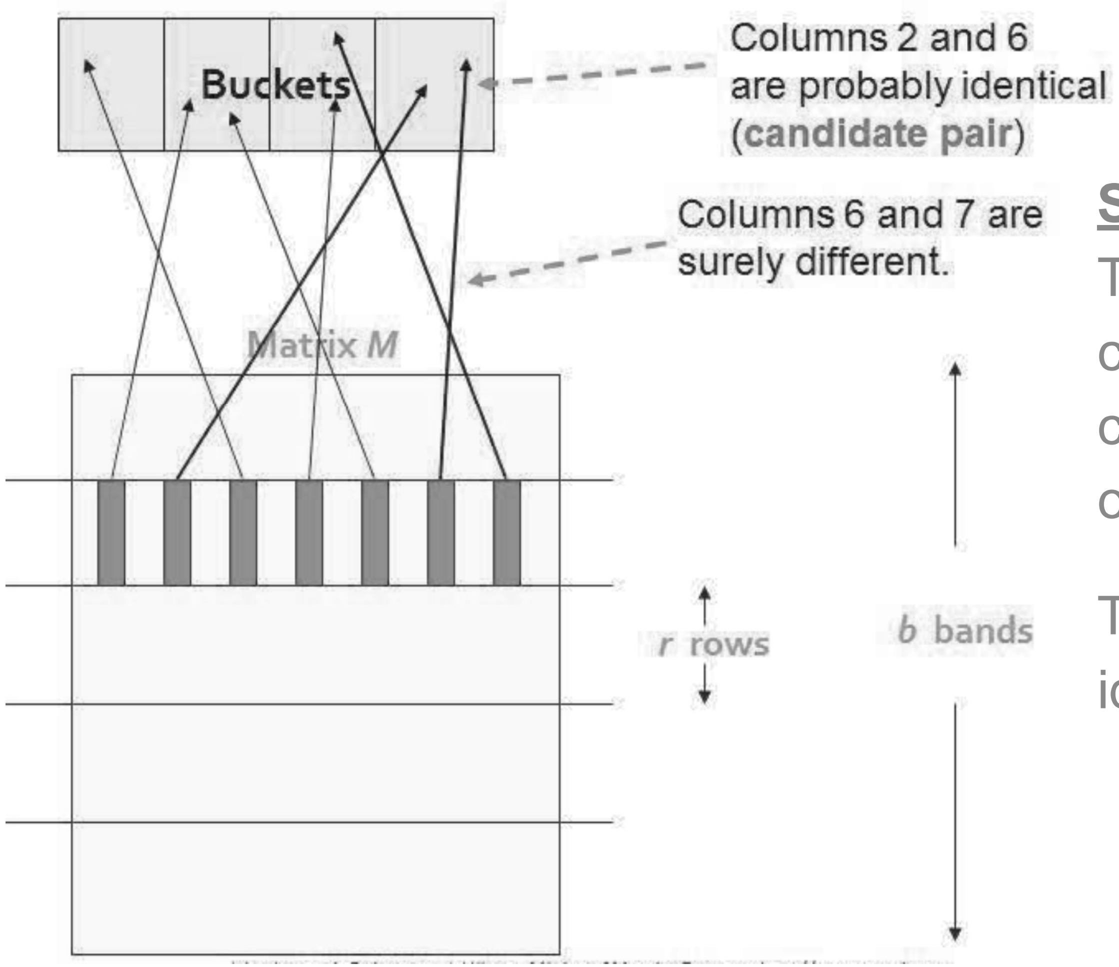



Criteria for being candidate pair:

- They end up in same bucket for at least 1 band.

(Leskovec at al., 2014; http://www.mmds.org/)

# Locality-Sensitive Hashing




**Step 1: Divide into *b* bands**
**Step 2: Hash columns within bands (one hash per band)**

**Simplification:**
There are enough buckets compared to rows per band that columns must be identical in order to hash into same bucket.

Thus, we only need to check if identical within a band.

(Leskovec at al., 2014; http://www.mmds.org/)

# Document-Similarity Pipeline

# Probability of Agreement

- 100,000 documents
- 100 random permutations/hash functions/rows

  => if 4byte integers then 40Mb to hold signature matrix

  => still 100k choose 2 is a lot (~5billion)

# Probability of Agreement

- 100,000 documents
- 100 random permutations/hash functions/rows

  => if 4byte integers then 40Mb to hold signature matrix

  => still 100k choose 2 is a lot (~5billion)
- 20 bands of 5 rows
- Want 80% Jaccard Similarity ; for any row $p(S_1 == S_2) = .8$

# Probability of Agreement

- 100,000 documents
- 100 random permutations/hash functions/rows
  => if 4byte integers then 40Mb to hold signature matrix
  => still 100k choose 2 is a lot (~5billion)
- 20 bands of 5 rows
- Want 80% Jaccard Similarity ; for any row $p(S_1 == S_2) = .8$

$P(S_1 == S_2 \mid b^{(5)})$: probability S1 and S2 agree within a given band

# Probability of Agreement

- 100,000 documents
- 100 random permutations/hash functions/rows
  => if 4byte integers then 40Mb to hold signature matrix
  => still 100k choose 2 is a lot (~5billion)
- 20 bands of 5 rows
- Want 80% Jaccard Similarity ; for any row $p(S_1 == S_2) = .8$

$P(S_1 == S_2 \mid b^{(5)})$: probability S1 and S2 agree within a given band

$= 0.8^5 = .328$

(Leskovec at al., 2014; http://www.mmds.org/)

# Probability of Agreement

- 100,000 documents
- 100 random permutations/hash functions/rows
  => if 4byte integers then 40Mb to hold signature matrix
  => still 100k choose 2 is a lot (~5billion)
- 20 bands of 5 rows
- Want 80% Jaccard Similarity ; for any row $p(S_1 == S_2) = .8$

$P(S_1==S_2 \mid b^{(5)})$: probability S1 and S2 agree within a given band

$= 0.8^5 = .328 \quad \Rightarrow \quad P(S_1!=S_2 \mid b) = 1-.328 = .672$

(Leskovec at al., 2014; http://www.mmds.org/)

# Probability of Agreement

- 100,000 documents
- 100 random permutations/hash functions/rows
  => if 4byte integers then 40Mb to hold signature matrix
  => still 100k choose 2 is a lot (~5billion)
- 20 bands of 5 rows
- Want 80% Jaccard Similarity ; for any row $p(S_1 == S_2) = .8$

$P(S_1{==}S_2 \mid b^{(5)})$: probability S1 and S2 agree within a given band

$= 0.8^5 = .328 \quad \Rightarrow \quad P(S_1 != S_2 \mid b) = 1 - .328 = .672$

$P(S_1 != S_2)$: probability S1 and S2 do not agree in any band

(Leskovec at al., 2014; http://www.mmds.org/)

# Probability of Agreement

- 100,000 documents
- 100 random permutations/hash functions/rows
  => if 4byte integers then 40Mb to hold signature matrix
  => still 100k choose 2 is a lot (~5billion)
- 20 bands of 5 rows
- Want 80% Jaccard Similarity ; for any row $p(S_1 == S_2) = .8$

$P(S_1{==}S_2 \mid b^{(5)})$: probability S1 and S2 agree within a given band

$= 0.8^5 = .328 \quad \Rightarrow \quad P(S_1 {!=} S_2 \mid b) = 1-.328 = .672$

$P(S_1 {!=} S_2)$: probability S1 and S2 do not agree in any band

$= .672^{20} = .00035$

(Leskovec at al., 2014; http://www.mmds.org/)

# Probability of Agreement

- 100,000 documents
- 100 random permutations/hash functions/rows
  => if 4byte integers then 40Mb to hold signature matrix
  => still 100k choose 2 is a lot (~5billion)
- 20 bands of 5 rows
- Want 80% Jaccard Similarity ; for any row $p(S_1 == S_2) = .8$

$P(S_1{==}S_2 \mid b)$: probability S1 and S2 agree within a given band
$= 0.8^5 = .328$ => $P(S_1 != S_2 \mid b) = 1-.328 = .672$

$P(S_1 != S_2)$: probability S1 and S2 do not agree in any band
$=.672^{20} = .00035$

What if wanting 40% Jaccard Similarity?

# Document-Similarity Pipeline